

T. Speer

28 April 2010



Outline



The Combinatorial Track Filter

- The Kalman Filter
- The combinatorial Kalman Filter

Robust algorithms

Results from CRAFT and the first collisions



Least square estimators



- The Kalman Filter is mathematically equivalent to a global Least-Square minimization (LS)
- LS estimators optimal when
 - model is linear
 - random noise Gaussian
- If the model is linear and random noise is Gaussian:
 - LS estimators are unbiased and have minimum variance
 - Residuals and pulls of estimated quantities are also Gaussian
 - > Objective function obeys a χ^2 distribution
- For non-linear models or non-Gaussian noise, LS still the optimal linear estimators





- Method to estimate the states of a dynamic system
 - Used in time-series analysis, signal processing
- Recursive
 - Track parameters estimated from a set of reconstructed hits
 - One hit added at a time, updating the state
 - No large matrices need to be inverted
- Track is described as discrete dynamic system
 - System equation:

$$x_k = f_k(x_{k-1}) + \delta_k$$

- Evolution of the track in the tracking detector
- $\rightarrow x_k$: State of the track on layer k
- f_k : track model, between layer (k-1) and k
- δ_k : Process noise, between layer (k-1) and k (multiple scattering)

$$<\delta_k>=0, \quad Cov[\delta_k]=Q_k$$





Measurement equation:

dependence of the observations on the local state

$$m_k = h_k(x_k) + \epsilon_k$$

 $\rightarrow m_k$: measurement in layer k

 $\rightarrow h_{\nu}$: measurement model

 $\succ \varepsilon_{\nu}$: measurement error

 V_{k} : Covariance of the measurement

$$<\epsilon_k>=0, Cov[\epsilon_k]=V_k=G_k^{-1}$$





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If track model is not linear, first-order Taylor expansion used:

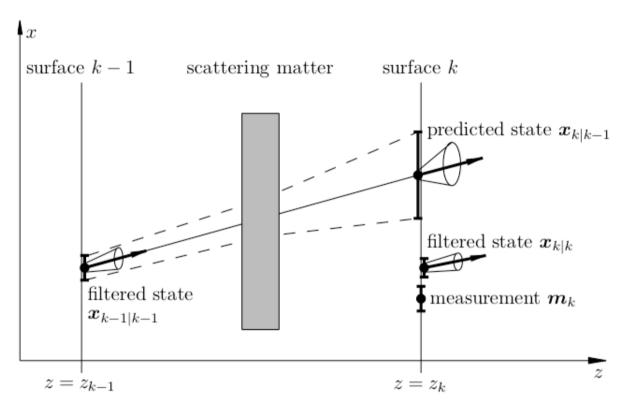
$$f_k(x_{k-1}) \approx f_k(x_e) + F_k(x_{k-1} - x_e) = F_k x_{k-1} + c_k$$

$$h_k(x_{k-1}) \approx h_k(x_e) + H_k(x_{k-1} - x_e) = H_k x_{k-1} + d_k$$

$$F_k = [\partial f_k / \partial x_k]_e$$
, $H_k = [\partial h_k / \partial x_k]_e$



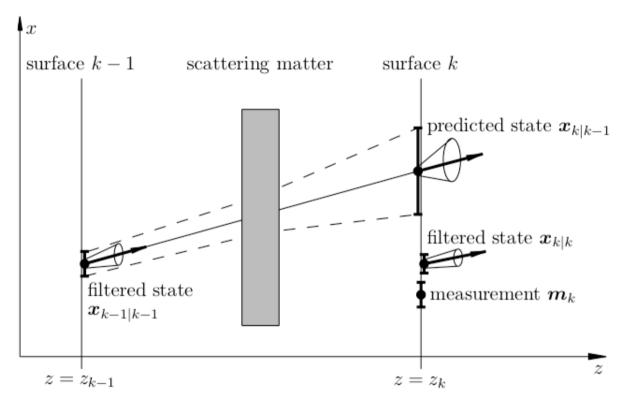




- Recursive procedure:
 - Prediction
 - Filtering
 - Smoothing





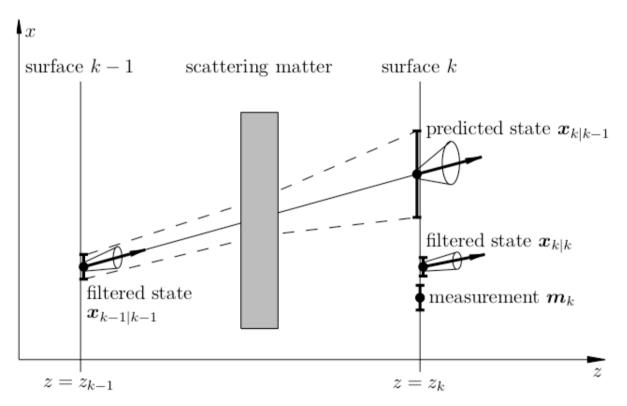


- Prediction: Trajectories are extrapolated from a layer to next layer, accounting for multiple scattering and energy loss
 - Prediction of state vector $x_{k|k-1}$, based on measurements $Y_{k-1} = \{y_1, ..., y_{k-1}\}$:

$$x_{k|k-1} = F_k x_{k-1|k-1}$$
, $C_{k|k-1} = F_k C_{k-1|k-1} F_k^T + Q_k$







- Filtering: On the new layer, new trajectories are constructed, with updated parameters (and errors) for each compatible hit in the layer.
 - Weighted mean of prediction and observation
 - > $x_{k|k}$, based on measurements $Y_k = \{y_1, ..., y_k\}$:

$$x_{k|k} = C_{k|k} [C_{k|k-1}^{-1} x_{k|k-1} + H_k^T G_k m_k]$$
, $C_{k,k} = [C_{k,k-1}^{-1} + H_k^T G_k H_k]^{-1}$





- Prediction: Trajectories are extrapolated from a layer to next layer, accounting for multiple scattering and energy loss
- Filtering: On the new layer, new trajectories are constructed, with updated parameters (and errors) for each compatible hit in the layer
- Smoothing: final fit of trajectories
 - Obtain optimal estimates at every measurement point along the track
 - In addition to providing tracks accurate at both ends this procedure provides more accurate rejection of outliers
 - Combination of forward and backward filters by a weighted mean



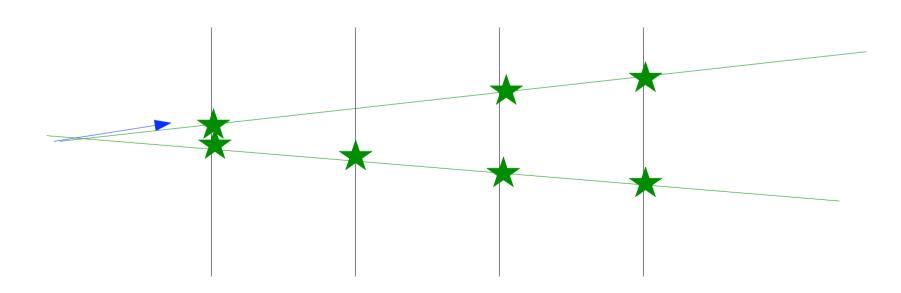


- Integrate track fitting and pattern recognition
- Build track(s) from an initial trajectory (seed)
 - Combinatorial exploration of all possibilities
 - Build all candidates in parallel to avoid bias





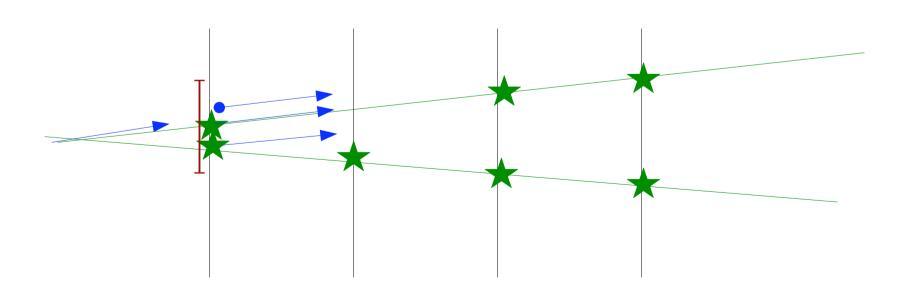
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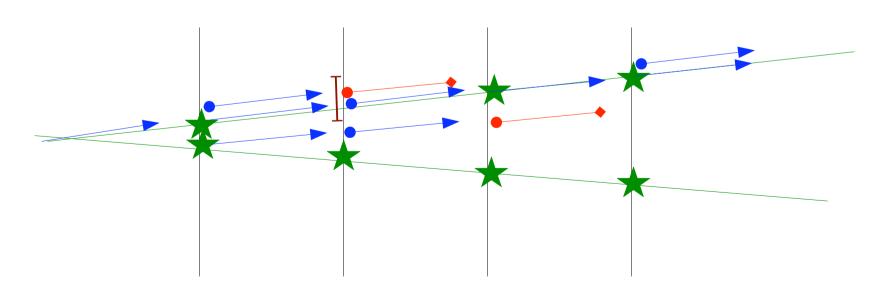






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- The number of trajectories to grow is limited according to their χ^2 and the number of missing hits







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- Since trajectories are grown in parallel, large number of duplicate and overlapping trajectories
 - Final collection has to be cleaned to remove duplicates





- Multi-step iterative Combinatorial Kalman Filter
- Decomposed in modular, independent, components:
 - Local reconstruction: hit reconstruction





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 - Trajectory building: construction of trajectories for a given seed
 - > Trajectory Cleaning: arbitration, duplicate tracks removed, based on number of shared hits and χ^2





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 - > Outlier rejection: reject hits due noise, δ -rays, nearby tracks
 - Rejection more efficient at this stage since final fit provides optimal estimates at every measurement point along the track
 - > Rejection based on χ^2 of the smoothed residual, pixel cluster probability
 - Recursive procedure: remove largest residual above threshold and refit





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 - Track selection:
 - Filter tracks that are likely fakes
 - Flag the expected expected of the tracks
 - $^{>}$ Based on normalized χ^2 , longitudinal and transverse impact parameters and significance

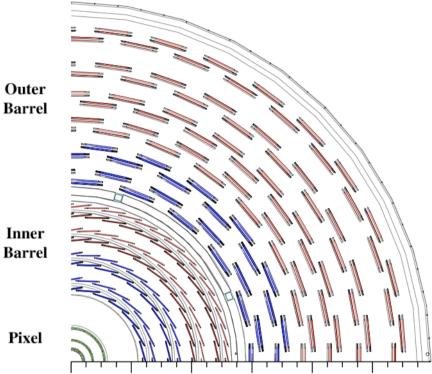


The CKF at CMS



Outside-in tracking:

- muon reconstruction: seeds in the outer layers based on muon-chamber seeds
- electrons from γ conversions: seeds in the outer layers based on ECAL clusters



Inner Barrel

Outer

Pixel

80

100 cm



The CKF at CMS



Outside-in tracking:

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Outer Barrel

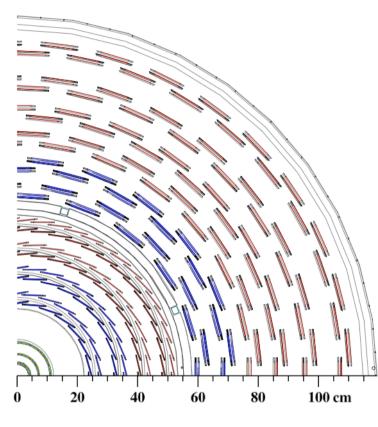
electrons from γ conversions: seeds in the B outer layers based on ECAL clusters

Inside-out tracking:

- Start in the first Pixel layers, grow tracks Inner layer by layer to the outer layer of the SST Barrel
- Favour tracks with pixel hits:
 - High precision

Pixel

- Important for further applications (vertex reconstruction, b-tagging)
- Fine granularity: low occupancy, high purity
- Reasonable number of seeds, with good quality
- Hadrons: nuclear interactions in the tracker, may not reach the outer layers
- Electrons loose energy because of bremsstrahlung radiation







Six-step iterative tracking sequence, with different seed types

Iteration	Seeding Layers	min p_T (GeV/c)	$\max d_{xy}$ (cm)
0	pixel triplets	0.8	0.2
1	pixel pairs	0.9	0.2
2	pixel triplets	0.075	0.2
3	pixel pairs	0.35	1.2
4	TIB/TID pairs	0.5	2.0
5	TOB/TEC pairs	0.8	5.0

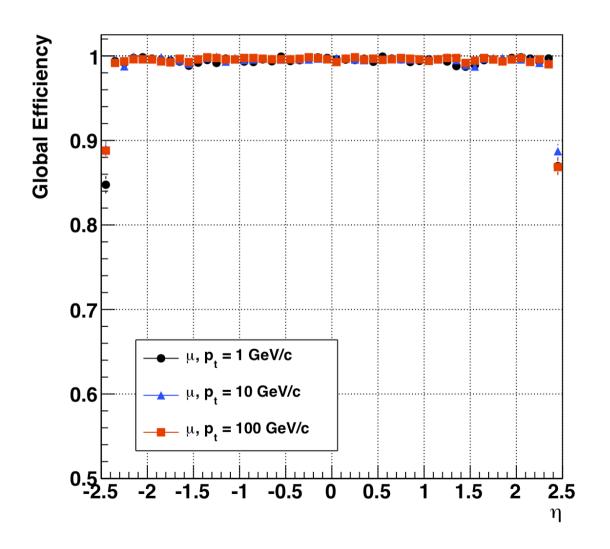
- After each iteration, remove hits unambiguously assigned to tracks
 - Next iteration will only use remaining hits
 - Allows to reduce the p_T threshold, Beam Spot compatibly
 - Recover tracks from V0, conversions, or for which the pixel detector was not fully efficient
- > In every iteration, run full CFK-sequence on available hits
 - Different (optimised) parameters used for each iterations

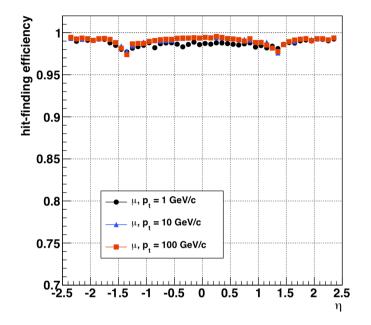


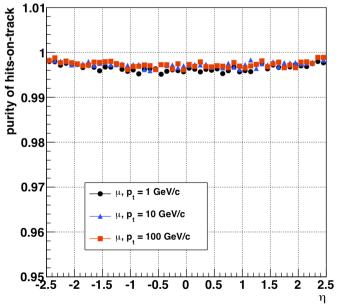


Track reconstruction for single tracks: muons, $p_T = 1$, 10, 100 GeV/c

in Monte Carlo





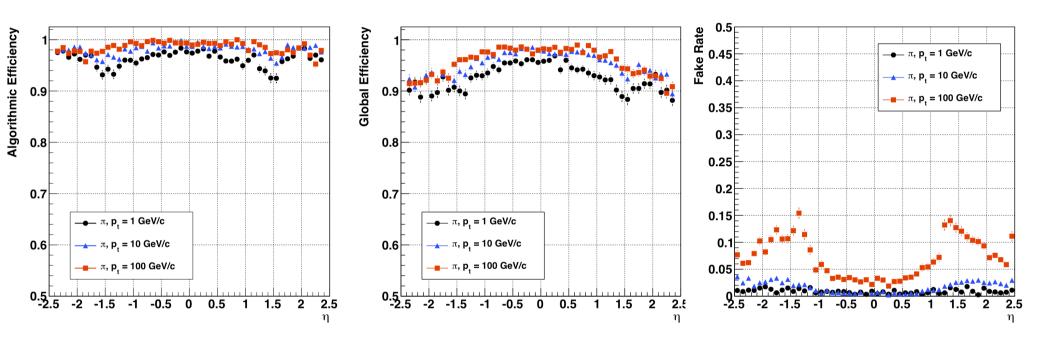






MC Track reconstruction for single tracks: pion, $p_{T} = 1$, 10, 100 GeV/c

- For pions: lower efficiency due to nuclear interactions in the tracker
- Algorithmic efficiency: for simulated tracks with at least 3 hits

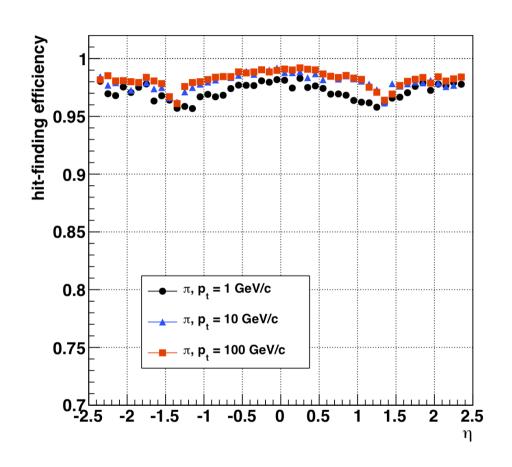


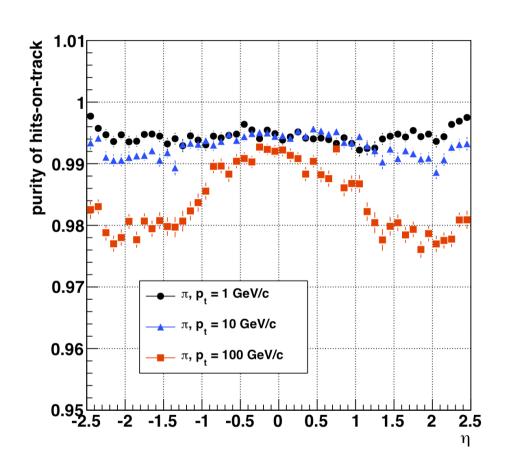
- Higher fake rate with higher energy: merging of primary and secondary particles:
 - Higher number of secondary particles
 - Smaller angle between the tracks, and smaller variation between the curvature of the primary and secondary particles.





MC Track reconstruction for single tracks: pion, $p_T = 1$, 10, 100 GeV/c

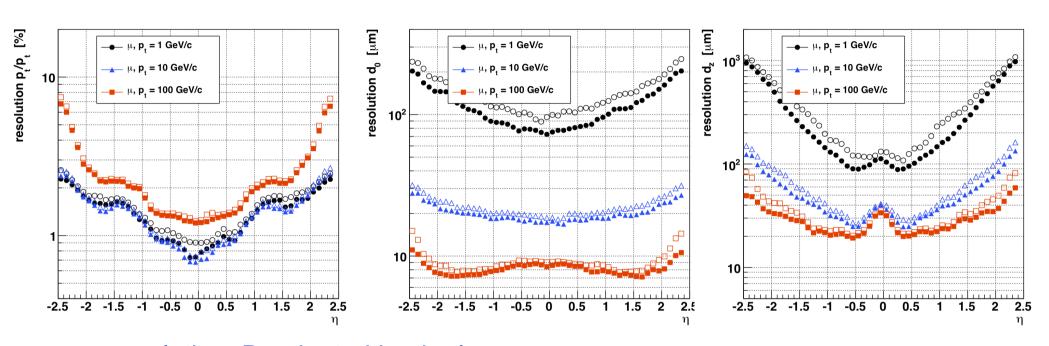








MC Track parameter resolution, single muons



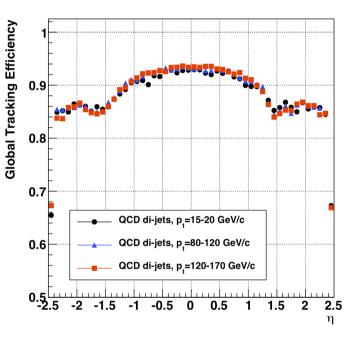
- \rightarrow p_T resolution: Dominated by the lever-arm
- Impact parameter resolution:
 - High momentum: Dominated by the resolution of the hits in the pixel
 - Low momentum degradation due to multiple scattering

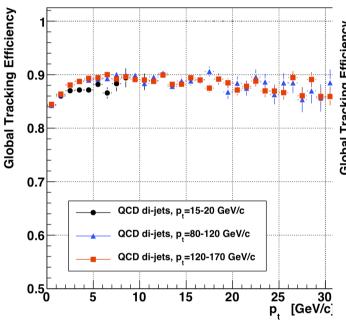


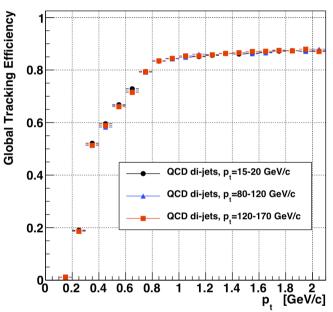


MC Track reconstruction for dijet events

includes underlying event, no Pile-Up

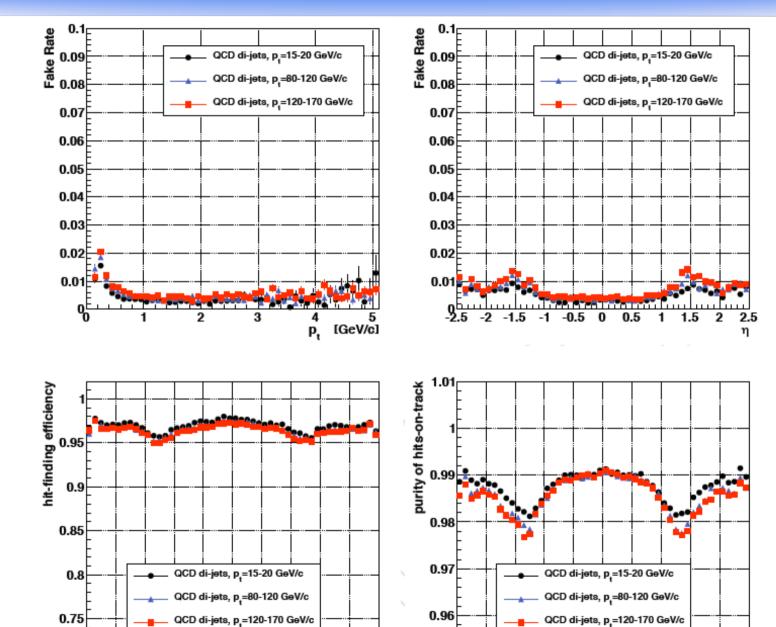












0.95

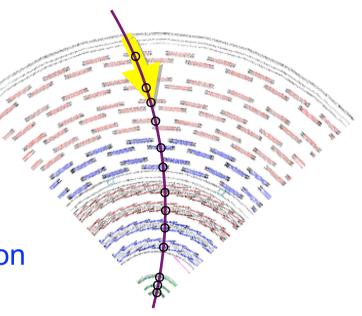


Track reconstruction in Cosmic Runs



In CRAFT runs, two algorithms used to reconstructs tracks:

- CTF: Default tracking algorithm modified for cosmic reconstruction:
 - Seeding in outer TOB and TEC layers
 - Outside-in pattern recognition with loose cuts
 - Track is reconstructed in the whole tracker
 - Reconstruction both downwards and upwards
- CosmicTF: dedicated cosmic reconstruction algorithm: 1 track/evt.
- Results from the algorithms are used as cross-check and to debug the reconstruction

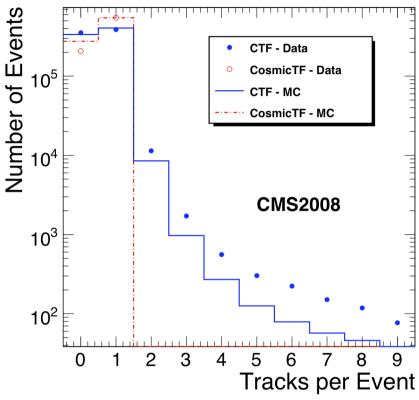




Track reconstruction in Cosmic Runs



Number of tracks per event



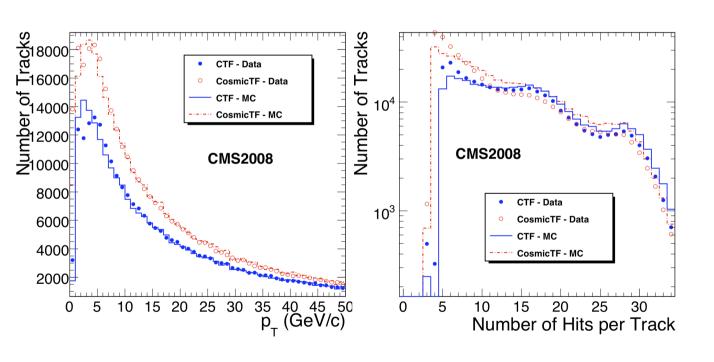
- Reconstruction of showers not optimal
- CTF not been optimised to reconstruct cosmic showers
- Multi-track events contain a number of fake or badly reconstructed tracks
- Mostly low momentum tracks with a small number of hits and large
- Fake rate is estimated to be around 1%.

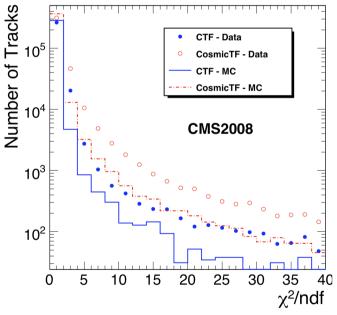


Track reconstruction in Cosmic Runs



- Track reconstruction parameters for 1-track events
- Small discrepancies:
 - Detector noise
 - Simulation in low momentum range of cosmic ray muons (e.g. position of the concrete plug covering the shaft)



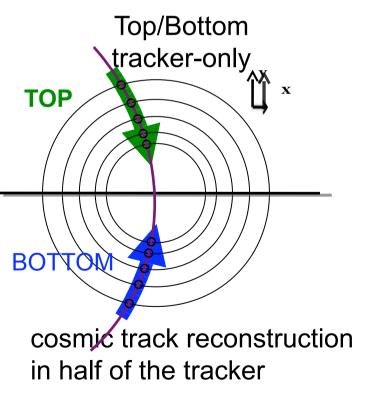


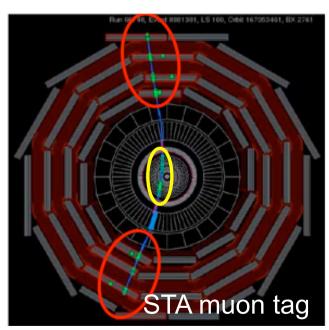


Track reconstruction efficiency in Cosmic Runs

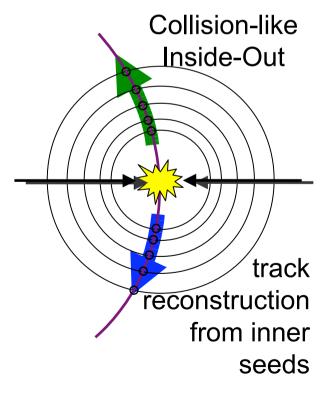


- 3 methods to estimate track reconstruction efficiency (barrel only):
 - 1) Tag (standalone muon) and probe (tracker muon), collision like method
 - 2) Tracker-only, independent reconstruction of top and bottom muon legs
 - 3) Standard inside out seeding for collisions and two legs matching





default cosmic track reconstruction

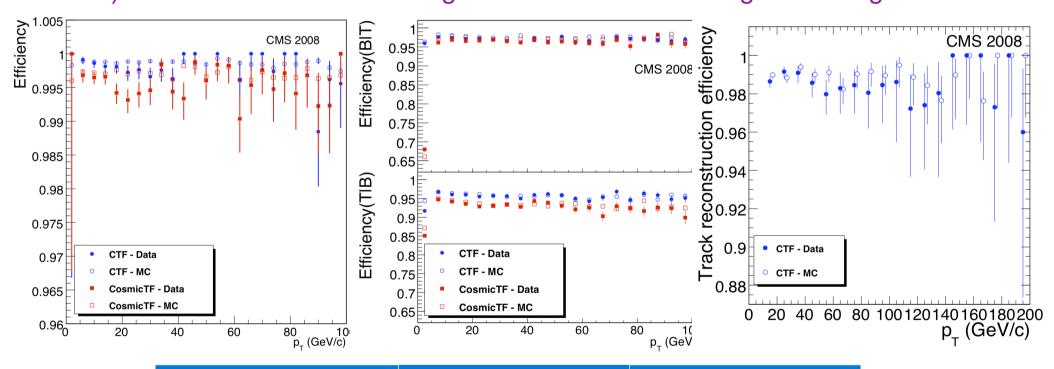




Track reconstruction efficiency in Cosmic Runs



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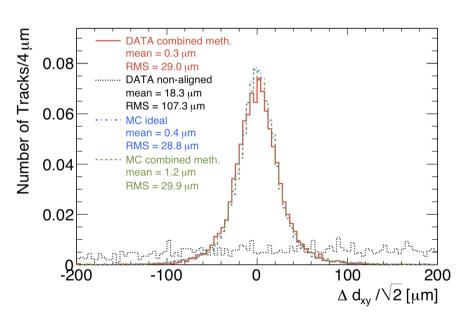
Collision-like reco:	Data	MC
Seeding	99.17 ± 0.12	99.30 ± 0.08
Pattern recognition	99.79 ± 0.06	99.64 ± 0.05
Overall	98.96 ± 0.13	98.94 ± 0.09

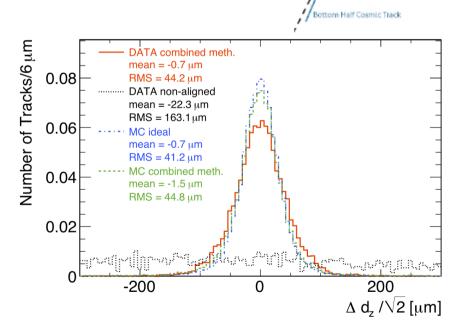


Track parameter resolution



- Splitting cosmic tracks along distance of closest approach to beamline
- Refit top and bottom halves and find the difference in the track parameters
- Validation of the alignment
 - Parameter resolution approaching those of a MC simulation with ideal detector geometry



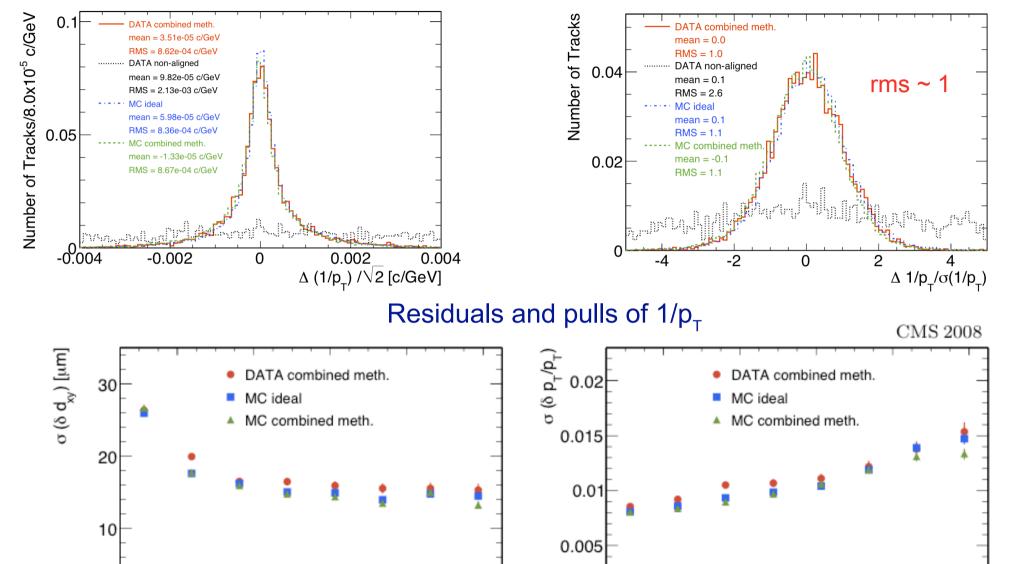


Residuals of transverse and longitudinal impact parameter



Track parameter resolution





RMS of Residuals versus p_T for d_{xy} and p_T

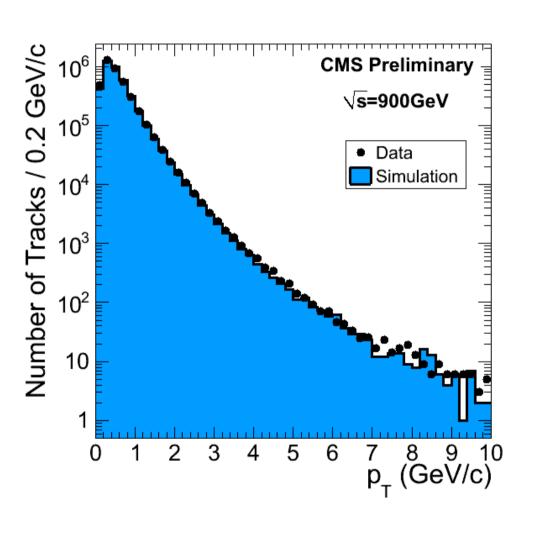
p_{_} [GeV/c]

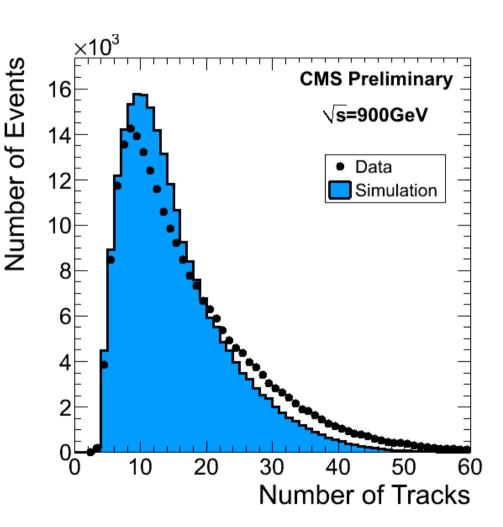


Track reconstruction in 900 GeV collisions



Track reconstruction in pp collisions at 900 GeV (December 2009):

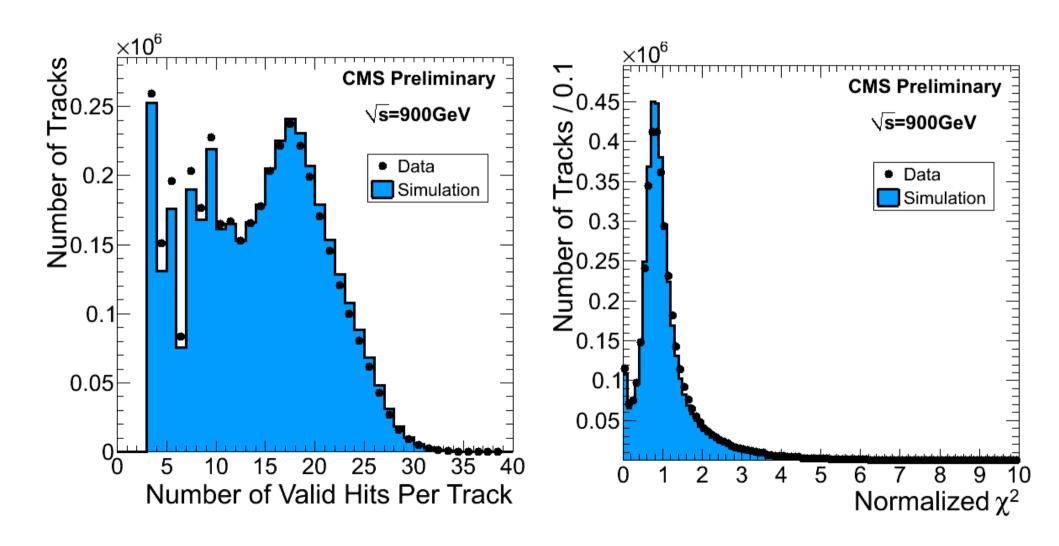






Track reconstruction in 900 GeV collisions

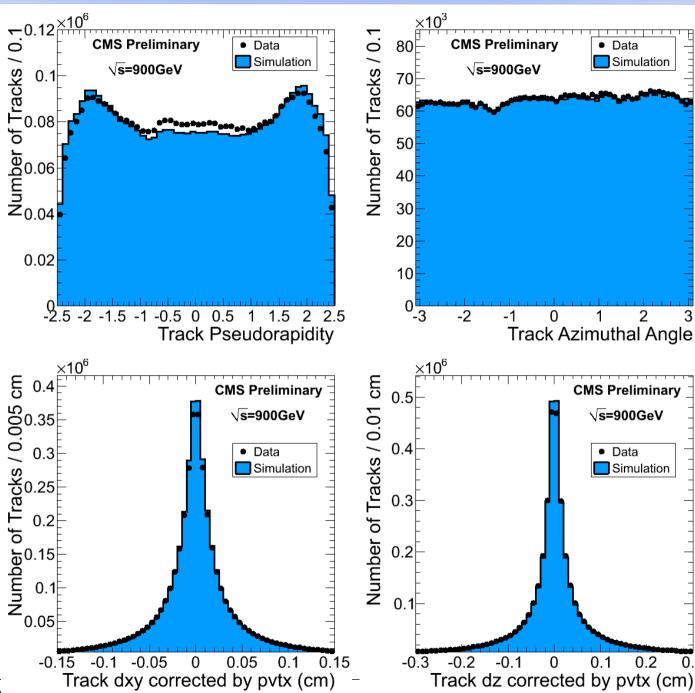






Track reconstruction in 900 GeV collisions





T. Speer: Track reconstruction in Civio

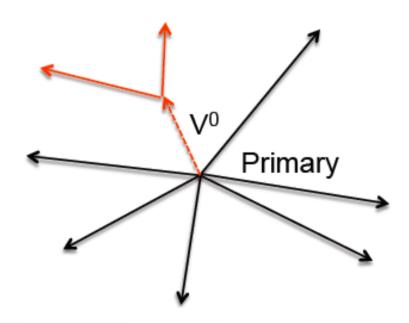
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Reconstruction of long-lived K⁰_s and Λ



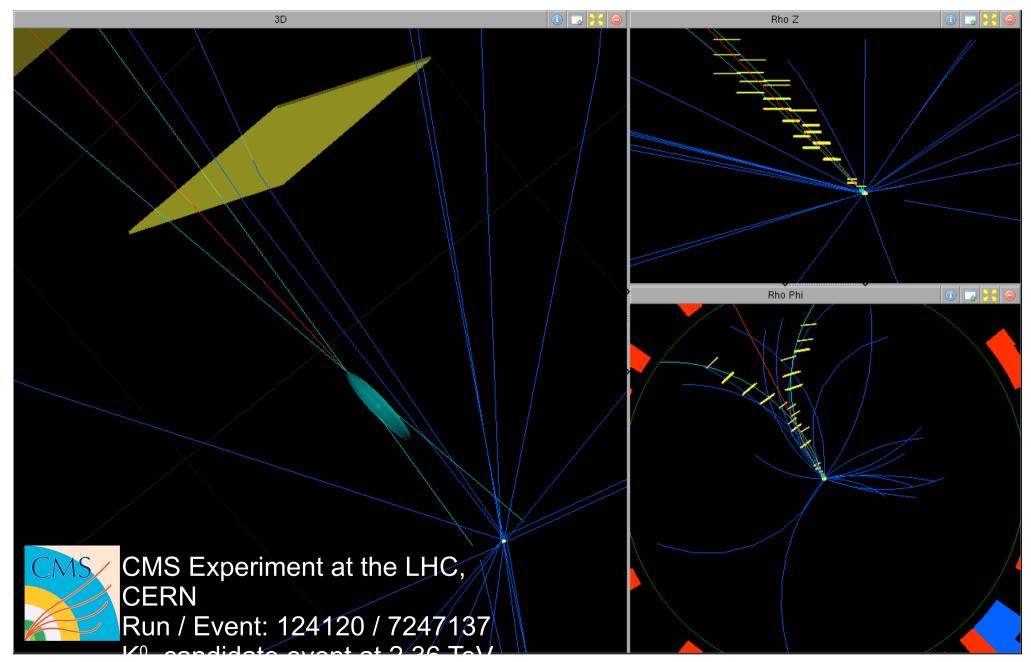
- Reconstruction of long-lived K_s^0 and Λ :
 - $K_s^0 \to \pi^+\pi^-$
 - $\Lambda \to p\pi^-$
- Selection requirements
 - High quality oppositely charged tracks
 - Tracks not compatible with primary vertex
 - Displaced decay vertex
 - No track hits before the secondary vertex





K⁰ candidate event at 2.36 TeV



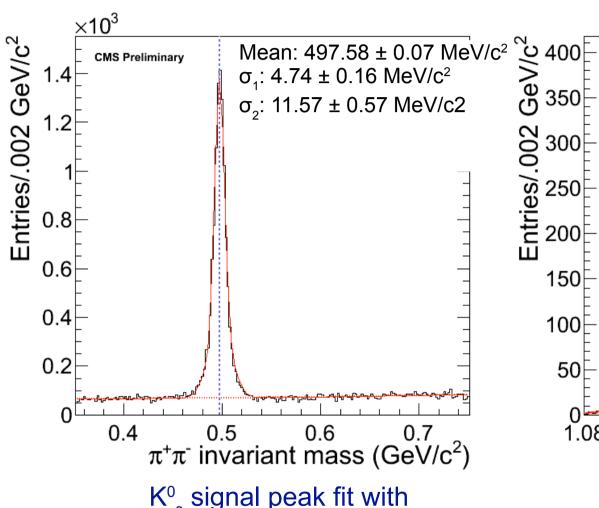




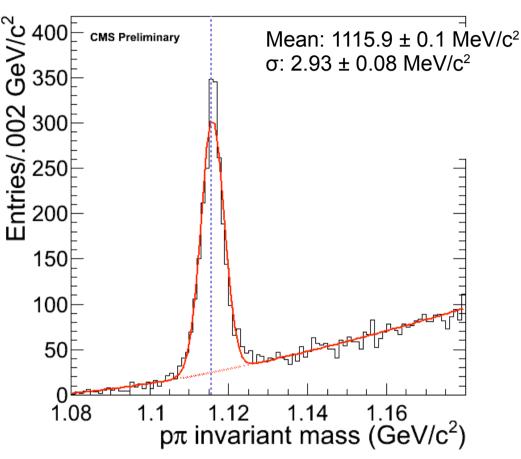
Reconstruction of long-lived K_s^0 and Λ



Invariant mass of K_s^0 and Λ candidates



K⁰_s signal peak fit with double Gaussian in good agreement with PDG mass (497.61 MeV/c²)



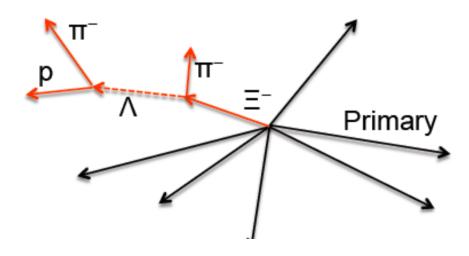
signal peak fit with single Gaussian in good agreement with PDG mass (1115.7 MeV/c²)

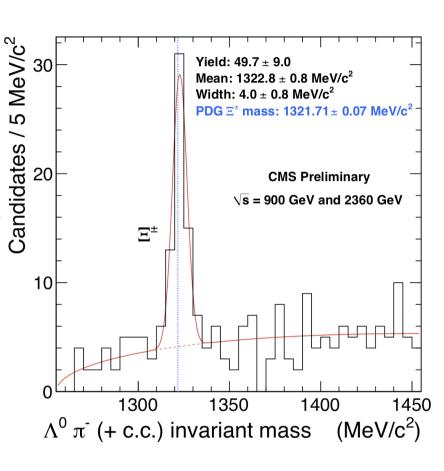


Reconstruction of E-baryon



- Reconstruction of ∃⁻ baryon
 - Reconstruction of Λ
 - Vertex fit of Λ candidate with charged track,
 - Mass of Λ candidate constrained to PDG mass
 - Tracks not compatible with primary vertex
 - No track hits before the secondary vertex ວ
- Clear mass peak consistent with E⁻ production





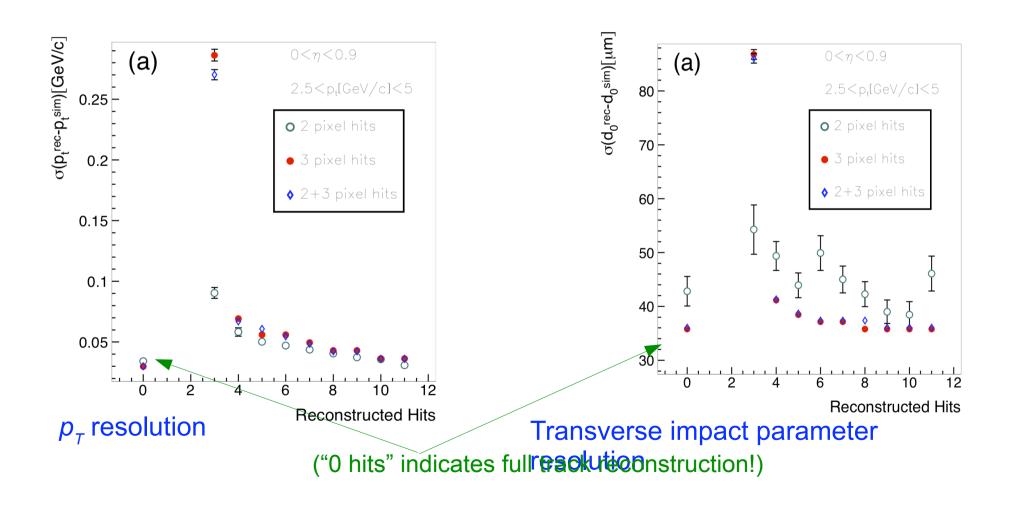


Partial reconstruction



- Combinatorial KF also suitable for usage in the High-Level Trigger (HLT):
 - Track parameter resolutions reach asymptotic value after using 5/6 hits

Resolutions as a function of the number of hits used: (b-jets, 2.5< p_T <5, $|\eta|$ <0.9)

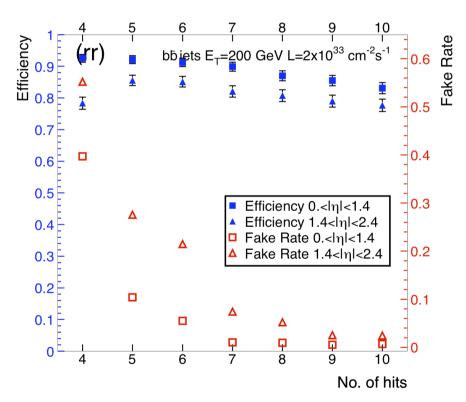




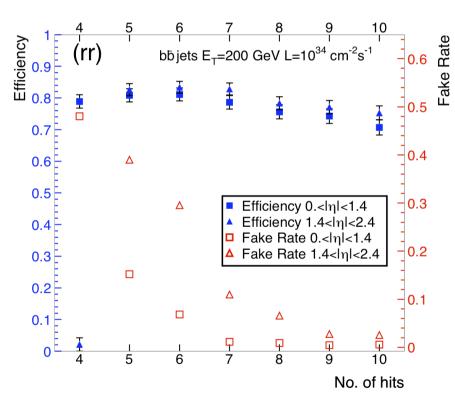
Partial reconstruction



- Partial reconstruction: stop track reconstruction once enough information is available to answer a specific question
- Same components, algorithms used.
- Precision sufficient for most HLT applications (b-tagging)



b-jets, E_{τ} = 200GeV, low lumi.



b-jets, E_{τ} = 200GeV, high lumi.



Adaptive methods

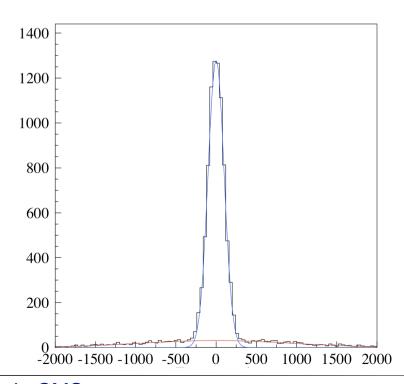


- Several non-linear algorithms have been implemented
- LSM optimal when
 - model is linear
 - random noise Gaussian
- Pdf involved are usually non-Gaussian:
 - Measurement errors have Gaussian core, with tails
 - Energy loss and multiple scattering (tails)
 - → Gaussian-sum Filter
- > Large background noise (electronic noise, low p_T tracks, δ electrons...)
 - Hit degradation
 - Hit assignment errors
 - → Deterministic Annealing Filter & Multi-Track Fit





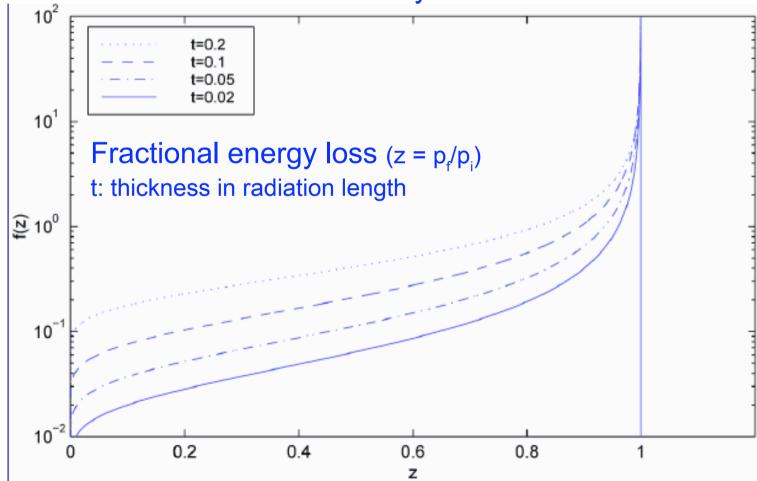
- Pdf involved are usually non-Gaussian:
 - Measurement errors have Gaussian core, with tails
 - Energy loss and multiple scattering (tails)
- Gaussian-sum Filter (GSF): instead of single Gaussian, model the pdfs involved by mixture of Gaussians:
 - Main component of the mixture would describe the core of the distribution
 - Tails would be described by one or several additional Gaussians.







- For electrons, above ~100MeV/c, energy loss dominated by bremsstrahlung
 - Bethe and Heitler energy loss model is highly non-Gaussian
 - > In the standard KF, distribution approximated by single Gaussian
- Model the Bethe-Heitler distribution by a mixture of Gaussians



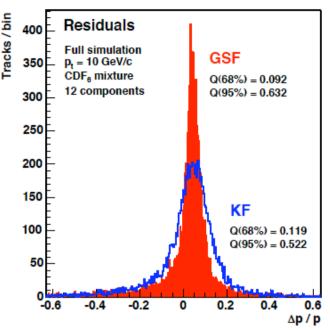


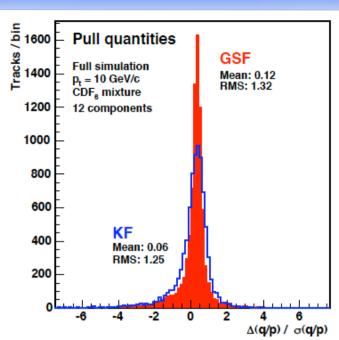


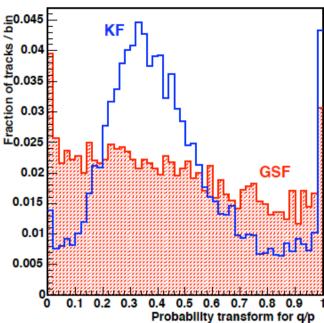
- All involved distributions are Gaussian mixtures
- State vector is also distributed according to a mixture of Gaussians
- GSF: Non-linear generalization of the Kalman Filter
- Weighted sum of several Kalman Filters
 - GSF is implemented as a number of Kalman filters run in parallel
 - The weights of the components are calculated separately
- Estimator is non-linear: weights depend on the measurements
- A pseudo- χ^2 can be defined, but it is not χ^2 distributed
- Exponential growth: combinatorial combination of the state vector components with energy-loss components
 - Number of components have to be limited to a predefined number at each step
 - Cluster (collapse) components with the smallest 'distance' (Distance measurements: Kullback-Leibler Distance or Mahalanobis Distance)
- Output is full Gaussian mixture of state vector
 - Can be used in subsequent application (GSF vertex fit)











- Improvement of the core of the residual distribution
- Little reduction of the tails:
 - Radiation in the first layer can not be detected
 - can be compensated by vertex constraint
 - Non-Gaussian measurement errors in the Pixel detectors
 - Incorporate Gaussian mixtures of measurement errors (also for non-e fits)
- Most efficient for low energy electrons (a few tens of GeV), little gain at 100 GeV

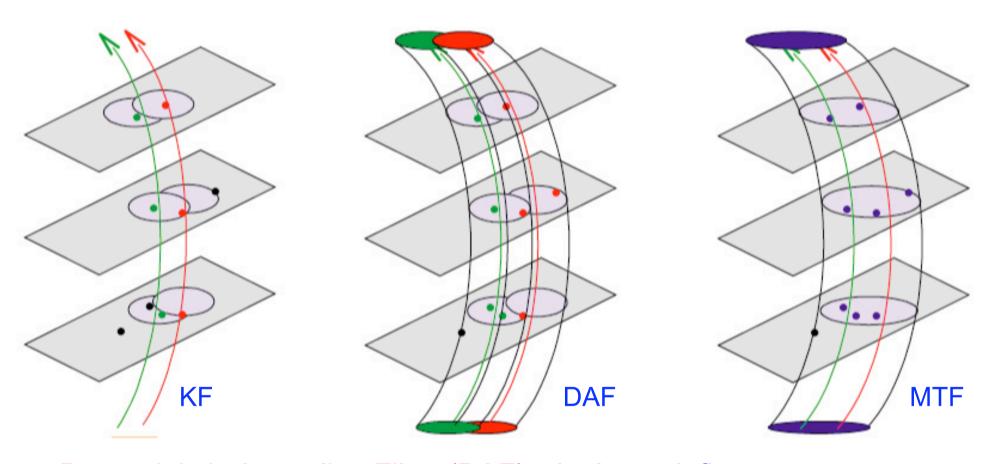




- In very dense environments (e.g. high E_{τ} b-jets, τ jets), degradation due to large background noise
 - High track density: hit degradation due to contamination of nearby tracks
 - High hit density: wrong hit assignment
- Kalman Filter: hard hit assignment
- Soft hit assignment may be more suitable
- Global approach of hit assignation, using full track information
 - Part of the hit assignment done in the final track fit





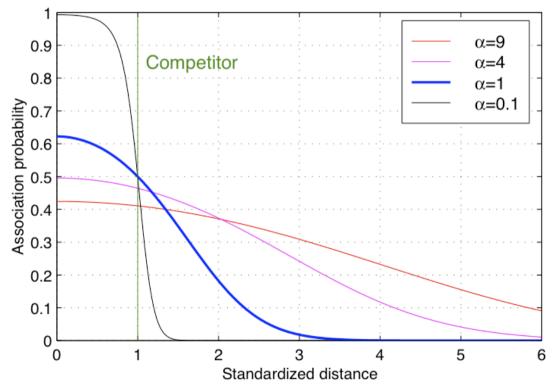


- Deterministic Annealing Filter (DAF): single track fit
 - On a same surface, several hits may compete for a track
- Multi-Track Fit (MTF): concurrent multi-track fit on collection of hits
 - Competition between tracks and hits
 - Each hit on a layer can belong to each of several tracks





- Deterministic Annealing Filter (DAF): single track fit
 - Competition between hits: on a same surface, several hits may compete for a track
 - Hit weights (assignment probability) based on hit-track distance (residual) and competing measurements



Hit weight in the presence of a competitor. The competitor is at one standard deviation from the track.





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 - Each hit on a layer can belong to each of several tracks
 - Hit weights based on hit-track distance and competing measurements and tracks
- Iterative Kalman Filter with annealing
 - Full Kalman fit (filter+smoother), using the current weights
 - Calculation of weights, using current estimates
 - The iteration ends when the weights are stable
- Deterministic Annealing helps to reach the optimal solution
 - Final assignment probabilities may depend on initial values
 - At the start T >> 1
 - At each iterations, T reduced according to a predefined schedule, until T = 1





- Both need initial hit collection and track seed(s): basic pattern recognition and track parameters from KF
 - DAF: initial hit collection around a KF track.
 - MTF: collection of tracks from KF (or even DAF), close in momentum space, hits collected around these tracks
- With this seeding, track finding efficiencies can not be improved w.r.t.
 KF

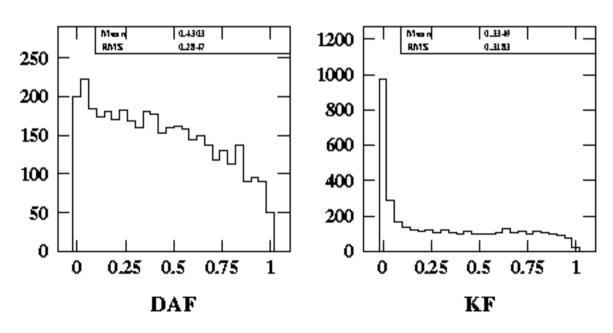


The Deterministic Annealing Filter



- For "isolated tracks", even at high luminosity, the DAF does not provide a measurable improvement in track quality
- "dense environment": b-jet with E_{T} =200 GeV,
 - > Tracks with $p_{\tau} > 15 \text{ GeV/}c$, min. 8 hits:
 - > Better track quality (χ^2)

χ^2 probability - $|\eta|$ <0.7

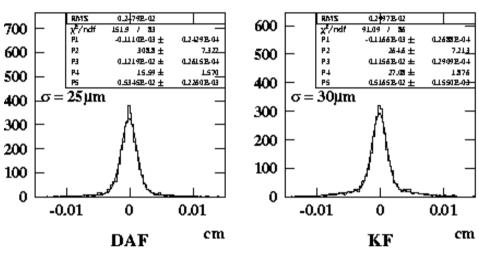




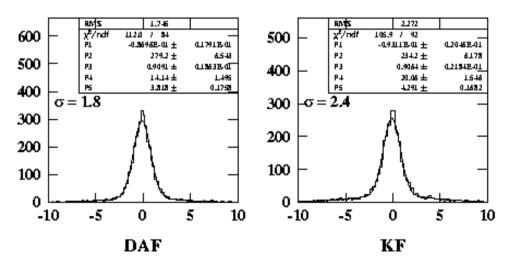
The Deterministic Annealing Filter

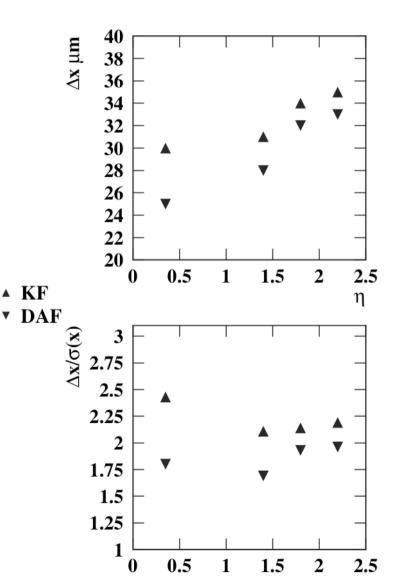


Transverse IP resolution - |η|<0.7



Transverse IP pull - |η|<0.7





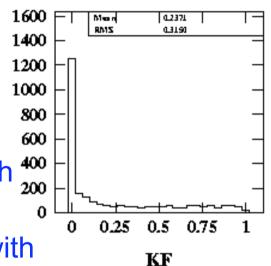
η

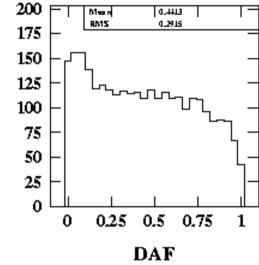


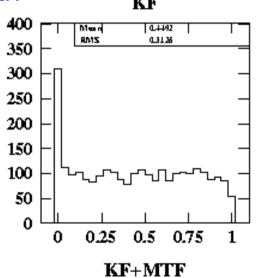


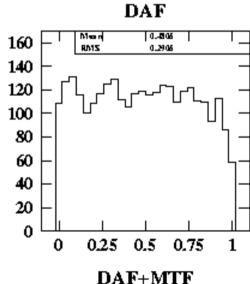
- Property Reconstruction of π tracks from the decay of high-p_{\tau} τ :
 - $H^0 \to \tau^+ \tau^-, m(H^0) = 500 \text{ GeV}/c^2$
- KF: Kalman Filter alone
- DAF: DAF with seed from KF
- KF+MTF: MTF tracks, seeded with KF tracks
- DAF+MTF: MTF tracks, seeded with
 DAF tracks

χ^2 probability





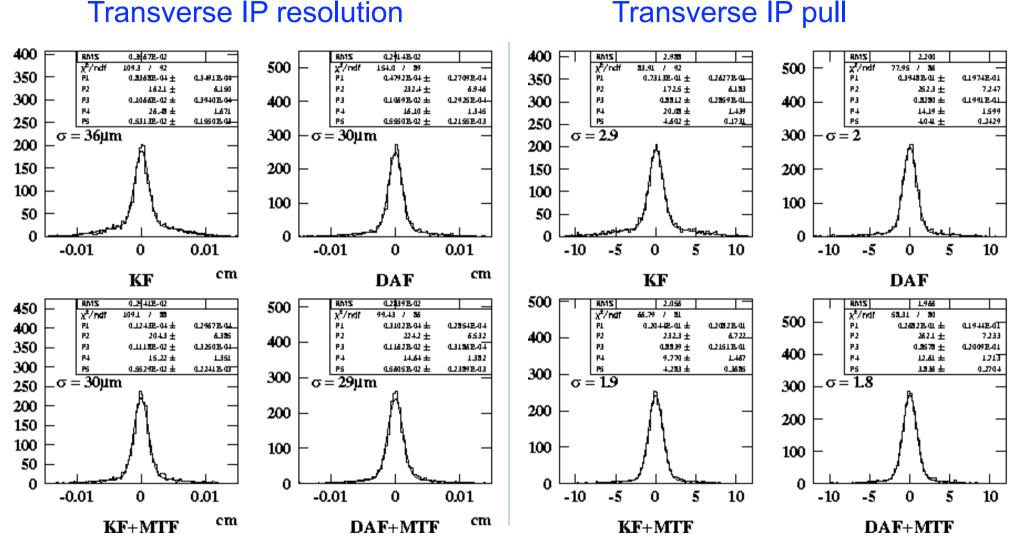








Transverse IP resolution



Little improvement with the MTF over the DAF





- For "isolated tracks", even at high luminosity, the DAF and/or the MTF do not provide a measurable improvement in track quality
- DAF: "dense environment", e.g. b-jet with ET=200 GeV , τ jets:
 - Better track parameter resolutions and error estimates
 - Better track quality (χ²)
- MTF: little improvement over DAF at the expected track densities!
 - little improvement on track parameter resolution
 - slightly better error estimate
 - slightly better overall track quality
- Better hit assignment (slightly lower fake rate)
- Seeding delicate (esp. MTF)
 - Better seeding methods would be needed
- Slower then standalone KF, use where appropriate



Conclusion



- A large number of algorithms have been implemented and are being evaluated
 - Several of these algorithms have never been tried before!
 - We have an array of tools to cope with different situations
- LSM shown to give very good (surprisingly good!) results even in difficult environments:
 - Well understood properties
 - Physicists know how to handle them and interpret the results.
- Combinatorial Kalman filter:
 - Efficient and robust pattern recognition
 - High efficiency, low fake rate
 - Low contamination from spurious hits, even with PU
 - Reconstruction ambiguities are solved after the first few layers
 - Good performance, suitable for high luminosity or heavy ion collisions
 - Fast enough for to be used in the HLT
 - Good track parameter resolutions after using only the first five to six hits
- Adaptive algorithms show improvements w.r.t. LSM in difficult situations





Backup



Alignment in CRAFT08



- Alignment strategy is based on running in chain two algorithms (16624 modules x 6 degrees of freedom):
 - local method on top of the geometry produced by global method
- Alignment with CRAFT08 better than expected

DMR RMS (µm)	Data Alignment	MC alignment	ldeal alignment	Modules >30 Hits
BPIX (x)	2.6	2.1	2.1	757/768
BPIX (y)	4.0	2.5	2.4	757/768
FPIX (x)	13.1	12.0	9.4	391/672
FPIX (y)	13.9	11.6	9.3	391/672
TIB (x)	2.5	1.2	1.1	2623/2724
TOB (x)	2.6	1.4	1.1	5129/5208
TID (x)	3.3	2.4	1.6	807/816
TEC (x)	7.4	4.6	2.5	6318/6400

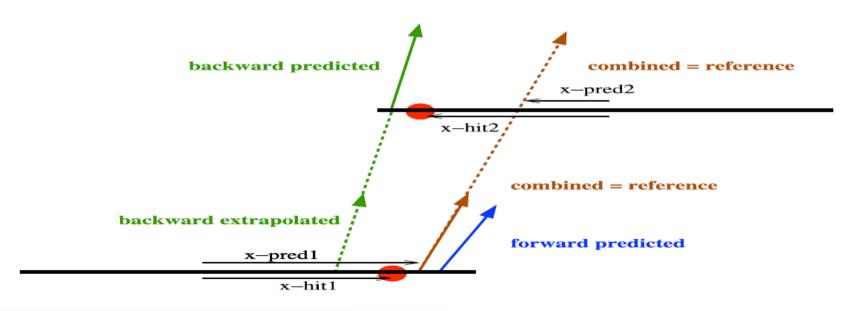
- Alignment validation
 - Track residuals and global parameters, sensor overlaps and geometry comparisons



Overlap study



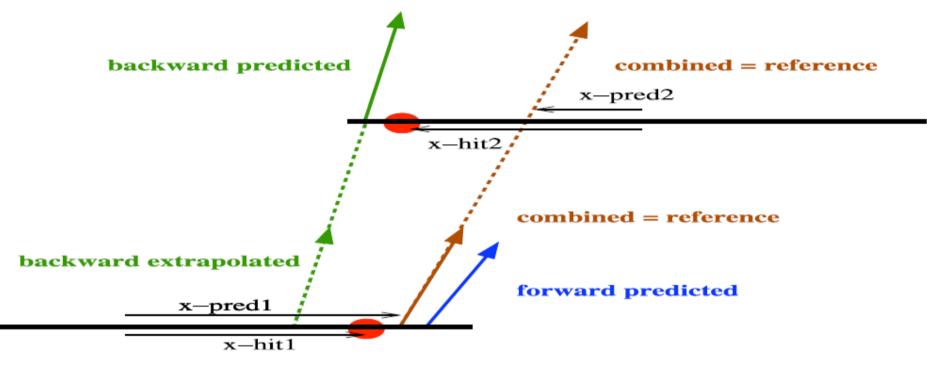
- Use overlaps to measure hit resolution, validate and monitor alignment
 - "Overlap" = reconstructed track crossing a layer in the overlapping part of two adjacent modules
- Using overlaps:
 - Reduces the amount of material between two layers
 - Reduces the effects of track extrapolation
 - Compare the predicted position from the track fitting at each module in the overlap with the position of the hits
- Asses relative displacement and rotations between adjacent modules





Overlap study





- Combine "forward predicted" and "backward predicted" tracks to give best possible track fit without using the hits in the overlap pair
- A better precision can be obtained by using the difference between the predicted position $\Delta x_{pred} = x_{1pred} x_{2pred}$, since it accounts for possible correlations between the modules
- Measure the accuracy of the prediction with the "double difference" $DD = \Delta x_{\text{pred}} \Delta x_{\text{hit}} \quad \text{(with } \Delta x_{\text{hit}} = x_{\text{1hit}} x_{\text{2hit}} \text{)}$



Track parameter resolution



Table 7: Standard deviation, mean, and 95% coverage of the residual and pull distributions of the track parameters. The units indicated pertain only to the residual distributions.

Track parameter	Residual distributions			Pull distributions		
	Std. Dev.	Mean	95% Cov.	Std. Dev.	Mean	95% Cov.
$p_T (\text{GeV}/c)$	0.083	0.000	1.92	0.99	0.01	2.1
Inverse p_T (GeV ⁻¹ c)	0.00035	0.00003	0.00213	0.99	-0.01	2.1
ϕ (mrad)	0.19	0.001	0.87	1.08	-0.02	2.4
θ (mrad)	0.40	0.003	1.11	0.93	-0.01	2.1
d_{xy} (μ m)	22	0.30	61	1.22	0.00	2.9
d_z (μ m)	39	0.28	94	0.94	-0.01	2.1